Four Major Asset Classes

Christopher Ting

http://www.mysmu.edu/faculty/christophert/

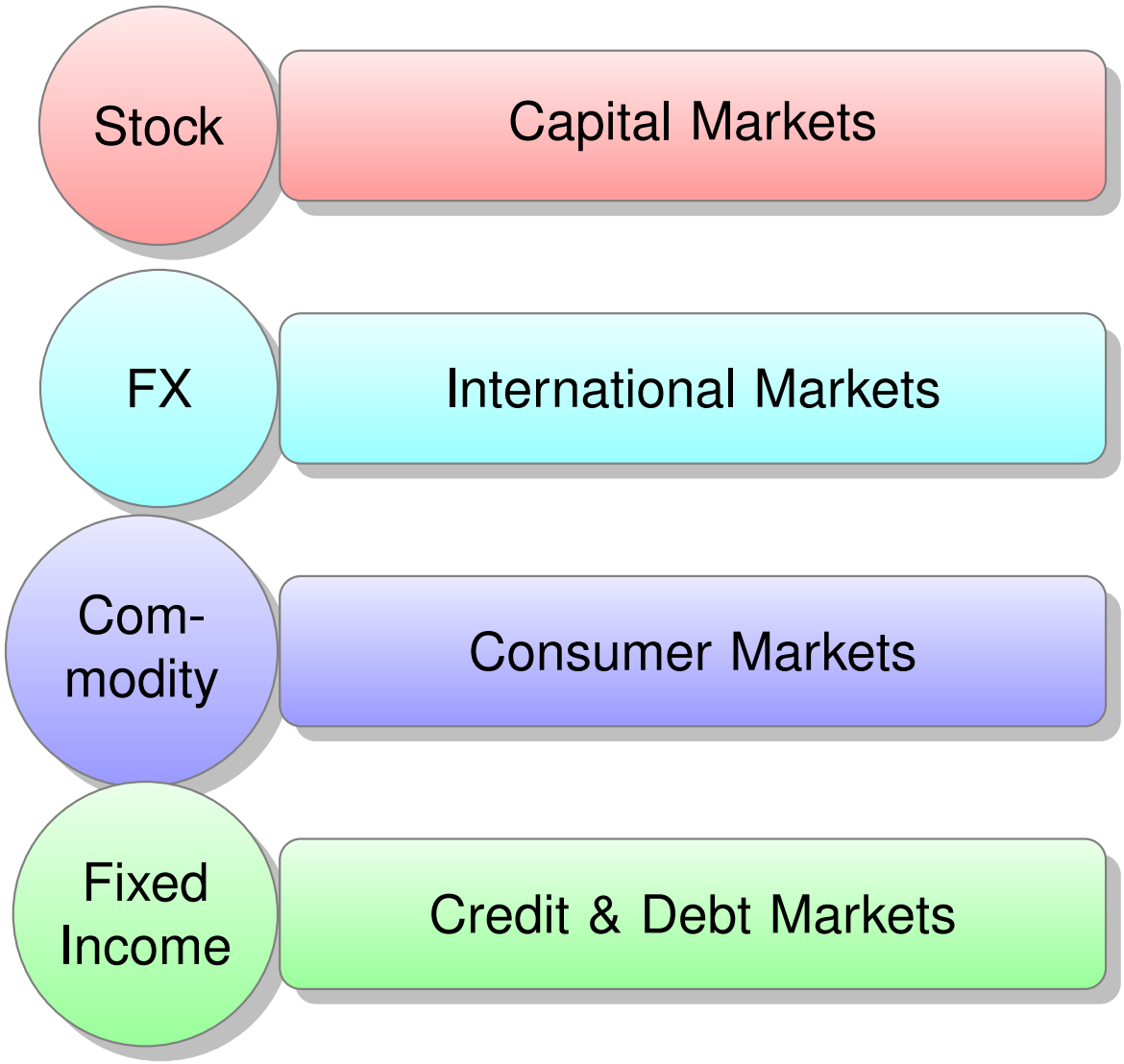
✉: christopherting@smu.edu.sg

☎: 6828 0364

: LKCSB 5036

Table of Contents

Underlying Assets in ℚuantitative Finance



Fundamentals of a Stock

- Ownership of a company: private equity versus public equity
- Investment in stock = investment in the company
- Claim on company's assets and earnings $\Longrightarrow$ dividend etc
- Limited liability: If the company goes bankrupt, shareholders walk away without losing personal assets. Important insight on stock price P_t:

$$P_t > 0 \tag{1}$$

- Transferability: Can be sold or bought in the secondary market $\Longrightarrow$ cash flow; liquidity

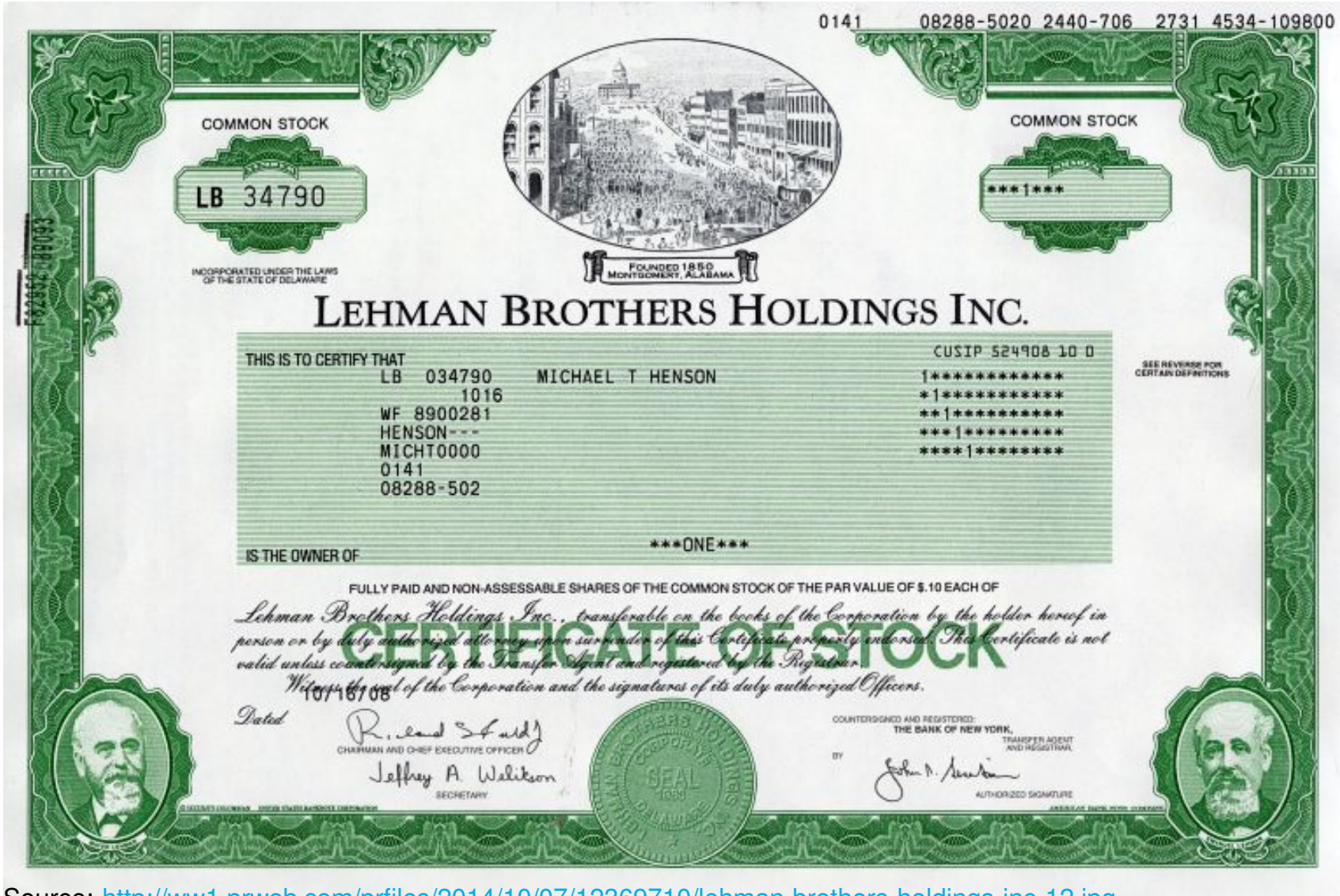
Stock Certificate: GE

Source: NYSE

Stock Certificate: Sony

Source: http://leeuwerck.blogspot.sg/2014/05/reading-japanese-dates-on-stock.html

Stock Certificate: Lehman Brothers



Source: http://ww1.prweb.com/prfiles/2014/10/07/12369710/lehman-brothers-holdings-inc-12.jpg

Time Series of Stock Prices

- Is there a good model for the prices P_t, $t = 0, 1, 2, \ldots, T$ of a given stock?
- Think about the return instead:

$$r_{t+1} := \frac{P_{t+1} - P_t}{P_t} \tag{2}$$

- Define a price change Δ_h over a time interval h:

$$\Delta_h := P_{t+h} - P_t \tag{3}$$

So the return is written as

$$r_{t+h} = \frac{\Delta_h}{P_t} \tag{4}$$

Time Series of Stock Prices (Cont'd)

- Model A: Return is the rate of return μ times the time interval h:

$$r_{t+h} = \frac{\Delta_h}{P_t} = \mu h \tag{5}$$

- As $h \to 0$, $h \to dt$, and $\Delta_h \to dP_t$ and we have

$$\frac{dP_t}{P_t} = \mu \, dt. \tag{6}$$

- To obtain P_t, we integrate from time 0 to time T. Applying your pre-U math, we obtain

$$\int_0^T \mu \, dt = \mu T \quad \text{and} \quad \int_0^T \frac{1}{P_t} dP_t = \ln(P_T) - \ln(P_0).$$

Time Series of Stock Prices (Cont'd)

- Since $\ln(P_T) - \ln(P_0) = \ln(P_T/P_0)$, we have

$$\ln\left(\frac{P_T}{P_0}\right) = \mu T, \tag{7}$$

which is

$$P_T = P_0 \exp\left(\mu T\right). \tag{8}$$

- Check for consistency: Does this model satisfy limited liability?
- Answer: Yes, because $e^{\mu T} > 0$ even if μ is negative and $T \to \infty$.
- Conclusion: Model A suggests that stock price increases at an exponential rate.

Tutorial

1. Exponential function

What are the properties of an exponential function?

$$f(x) = A\exp(bx), \quad \text{where } A \text{ and } b \text{ are constants}$$

2. Notation of a variable is dummy

What is the stock price P_t at time t, where $t \leq T$?

3. One time step apart

What is the stock price P_t at time t in terms of P_{t-1}?

Arithmetic versus Geometric Returns

The end-of-year S&P 500 levels for 2004 till 2015:

Year	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
Level	1,211.92	1,248.29	1,418.30	1,468.36	903.25	1,115.10	1,257.64	1,257.60	1,426.19	1,848.36	2,058.90	2,043.94
r_1		3.00%	13.62%	3.53%	-38.49%	23.45%	12.78%	0.00%	13.41%	29.60%	11.39%	-0.73%

- There are 11 one-year returns r_1
- Arithmetic average return is the summation of these 11 one-year returns divided by 11, which is 6.51%
- Geometric average return r_g is the result of solving the equation

$$(1 + r_{2005})(1 + r_{2006}) \cdots (1 + r_{2015}) = (1 + r_g)^{11}.$$

The result is $r_g = 4.87\%$.

Arithmetic versus Geometric Returns (Cont'd)

- The standard deviation of r_1 is 17.67%, and the standard error (se) is

$$\text{se} = \frac{17.67\%}{\sqrt{11}} = 0.053276.$$

- The t statistic under the null hypothesis of zero annual return is

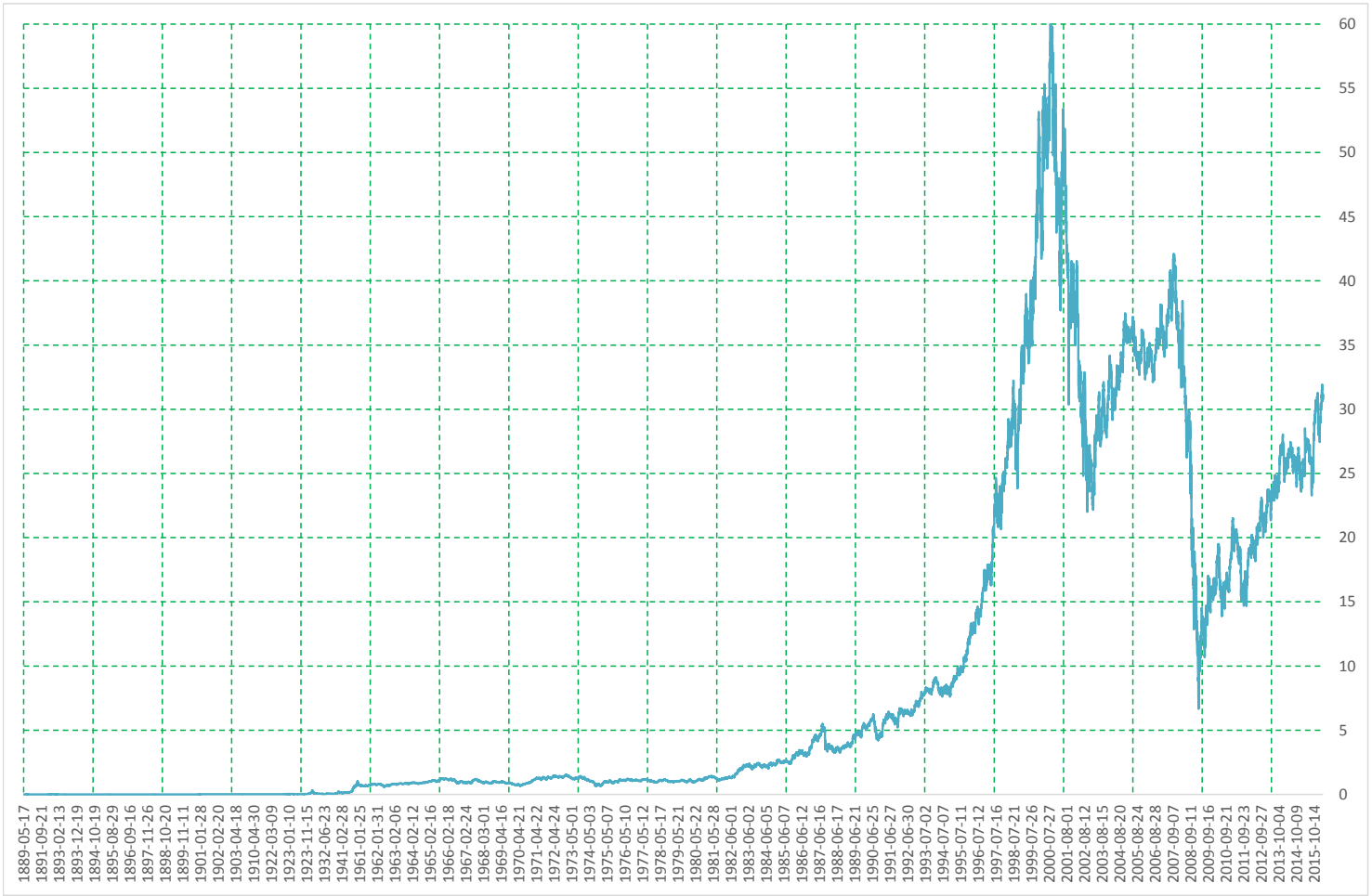
$$t = \frac{6.51\% - 0}{\text{se}} = 1.22$$

- Note that

$$\left(\frac{\text{S\&P}_{2015}}{\text{S\&P}_{2004}}\right)^{\frac{1}{11}} - 1 = \left(\frac{2043.94}{1211.92}\right)^{\frac{1}{11}} - 1 = 4.87\%.$$

- Which return is more realistic for an investor?
- S&P 500 index is just a number. Is there a way to invest indirectly in S&P 500 portfolio?

Data: Common Stock Prices of GE



Data source: Global Financial Data Sample period: 1889-05-17 to 2016-04-22

Fluctuations

- Clearly, Model A of stock prices is inadequate: it is deterministic and the price curve is too smooth.
- Model B: Fluctuation at time t is a realization of a random variable X_t:

$$P_t = P_0 \exp\big(\mu t\big) \times \exp\big(\sigma X_t\big) \tag{9}$$

i.e., with σ being a constant, multiply Model A by $e^{\sigma X_t}$, leading to

$$P_t = P_0 \exp\big(\mu t + \sigma X_t\big) \tag{10}$$

- Important implication of Model B: The stock price P_t is random!
- What should be the distribution of X_t?

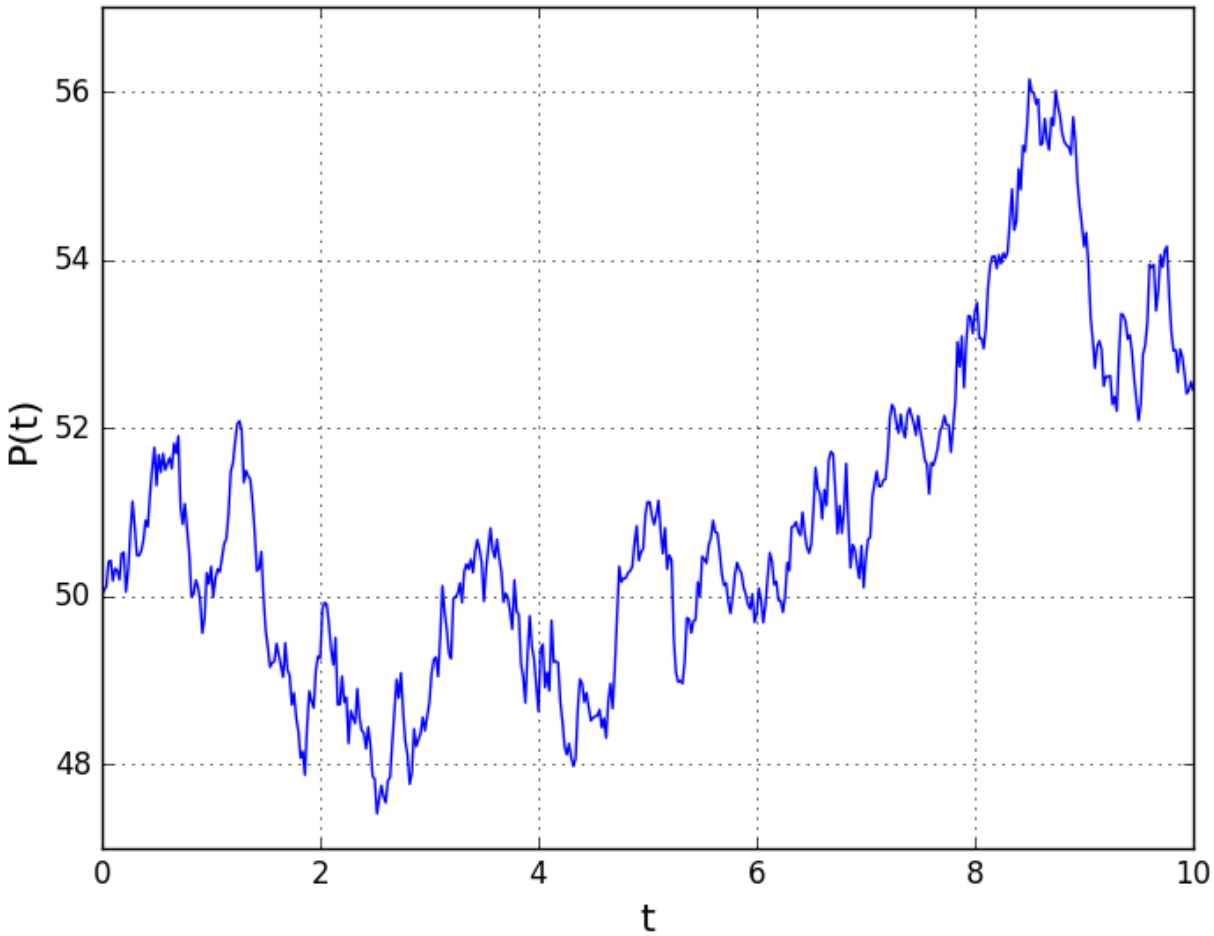
Practice for Stock Price: Geometric Brownian Motion

- For each time t, let Z_t denote the random variable of a standard normal distribution (mean 0 and variance 1).
- Model B-1: Geometric Brownian Motion (GBM)

$$\sigma X_t := \sqrt{\sigma^2 t}\, Z_t \tag{11}$$

  i.e., $X_t = \sqrt{t}\, Z_t$
- Why $\sqrt{t}$? You will see later in this course!
- Finance: GBM is used in simulation of stock prices, for which μ is called the rate of return, and σ the (rate of) volatility.
- The variance $\sigma^2 t$ is more natural in math but somehow market participants don't prefer it over volatility.

A Simulation of Geometric Brownian Motion



- $P_0 = 50$
- $\mu = 0$
- $\sigma = 2.0$

All Models Are Wrong!

All models are wrong
but some are useful.

Picture source: Wikipedia

Limitations

"In both social and natural sciences, ... there is no way to have a self-contained closed system or to avoid interaction between the observer and the observed. The Gödel theorem in mathematics, the Heisenberg uncertainty principle in physics, the self-fulfilling or self-defeating prophecy in the social sciences all exemplify these limitations."

Picture source: C250

Milton Friedman

Inflation and Unemployment

(1976), Page 348.

Fundamentals of FX

- A pair of currencies: buying one currency by selling another currency; long-short trading!
- Same as stock trading: buying one company's shares by selling cash
- Quote-driven FX market:

  Base Currency / Quote Currency

- One unit of base currency for x units of quote currency. Example: One US dollar for 1.3567 Singapore dollars

FX Example

- Suppose you are an investor in Singapore, and you think that the Japanese yen will appreciate against the Singapore dollar.
- Buy 1 million Japanese yens @ ¥100 against S$1.4500.
- How much do you pay in SGD?
  Answer:_____ (0 decimal in S$, e.g., S$1,234)
- In a sense, you buy 10,000 “lots” of JPY in units of 100 yens (“shares”).
- A month later, the exchange rate becomes ¥100 for S$1.5200.
- The Japanese yen has appreciated against the Singapore dollar. True or false?

Simple but Important Concepts in Finance

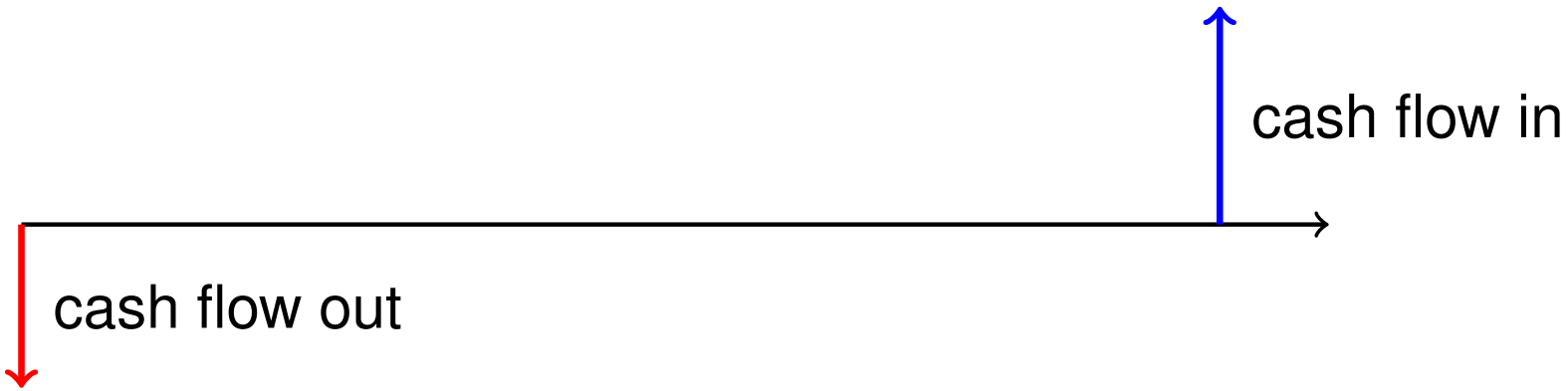
- Profit and Loss (P&L) is given by

$$\begin{aligned} \text{P\&L} &:= \text{Shares} \times \left(P_{t_\bigstar} - P_{t_\blacklozenge}\right) \\ &:= \text{Shares} \times \left(\text{selling price} - \text{buying price}\right) \end{aligned} \tag{12}$$

- It does not matter whether you buy first ($t_\bigstar > t_\blacklozenge$), or you sell first ($t_\bigstar < t_\blacklozenge$).
- Return (over a period of time) R is

$$\begin{aligned} R &:= \frac{\text{cash flow in} - \text{cash flow out}}{\text{cash flow out}} \\ &= \frac{\text{selling price} - \text{buying price}}{\text{buying price}} \end{aligned} \tag{13}$$

Proof of Return Formula



Proof:

- The cash flow out to buy N shares of the asset is $N \times P_{t_\blacklozenge}$.
- So $N \times P_{t_\blacklozenge}$ is the cash required for investment. You must have this capital before you can even invest.
- The cash flow in (received) for selling N shares is $N \times P_{t_\bigstar}$.
- The return is on (with respect to) the capital, given the P&L.
- Hence N in the denominator and the numerator cancels out and we obtain Equation (13).

Tutorial

1. P&L

Continuing from Slide 21, what is your P&L if you sell your holding of Japanese yens?

$$¥1{,}000{,}000 \times \big(0.0152 - 0.0145\big) = \text{S\$}700.00.$$

2. Return

Continuing from Question 1, what is the return you have realized over a month?

$$\frac{0.0152 - 0.0145}{0.0145} = 4.8276\%.$$

Market Practice

- The market practice has it that USD is always the base currency except
  - Euro: EUR
  - British Pound: GBP
  - Australian Dollar: AUD
  - New Zealand Dollar: NZD

- Interestingly, dealers trade these currencies by their nicknames: Fiber for EUR, Sterling for GBP, Aussie for AUD, and Kiwi for NZD. The U.S. dollar is nicknamed the Greenback or Buck, Swiss franc the Swissy, Canadian dollar the Loonie, and so on

Short Selling

- Short selling is to sell an asset that you don't already have, with a view that you will buy it back when the price is lower.
- Not so easy to short sell stocks (due to many constraints and costs) but very natural with FX.
- Example: Suppose you anticipate the British pound to depreciate against the Singapore dollar. Though you do not have any British pound, you can easily "short sell" GBP through your online forex trading account.

Tutorial

- Suppose a pound is selling for S$2.0500 and you short sell £10,000 to obtain S$20,500.
- A month later, suppose the Sterling becomes S$1.9900 per pound. You buy back the British pound at this rate. vfill
- The selling price is S$$1/1.99$ in this transaction and the buy-back price is S$$1/2.05$.
- In this example, the transaction volume is S$20,500 because the buying and selling rates are in pounds per Singapore dollar. Hence, the P&L is, in pounds

$$\text{P\&L} = \text{S\$}20{,}500 \times \left(\frac{1}{1.99} - \frac{1}{2.05} \right) = \text{£}301.51.$$

Tutorial (Cont'd)

- Correspondingly, the return realized in British pounds is

$$\frac{\frac{1}{1.99} - \frac{1}{2.05}}{\frac{1}{2.05}} = 3.0151\%$$

- What about the P&L in Singapore dollars?
  - Short selling GBP is equivalent to buying SGD. Recognizing this fact, the trade volume is no longer S$20,500 but £10,000.
  - So, the P&L is

$$£10{,}000 \times (2.05 - 1.99) = S\$600.$$

- The return over one month is

$$\frac{2.05 - 1.99}{1.99} = 3.0151\%.$$

A Subtlety in FX Calculation

- As shown earlier, the P&L in pounds is £301.51 and the amount of £10,000 is involved. The ratio of these two numbers in pounds is 3.0151%.
- However, in Singapore dollar terms, the P&L of S$600 divided by S$20,500 is 2.9268%.
- Which rate of return is correct and why?

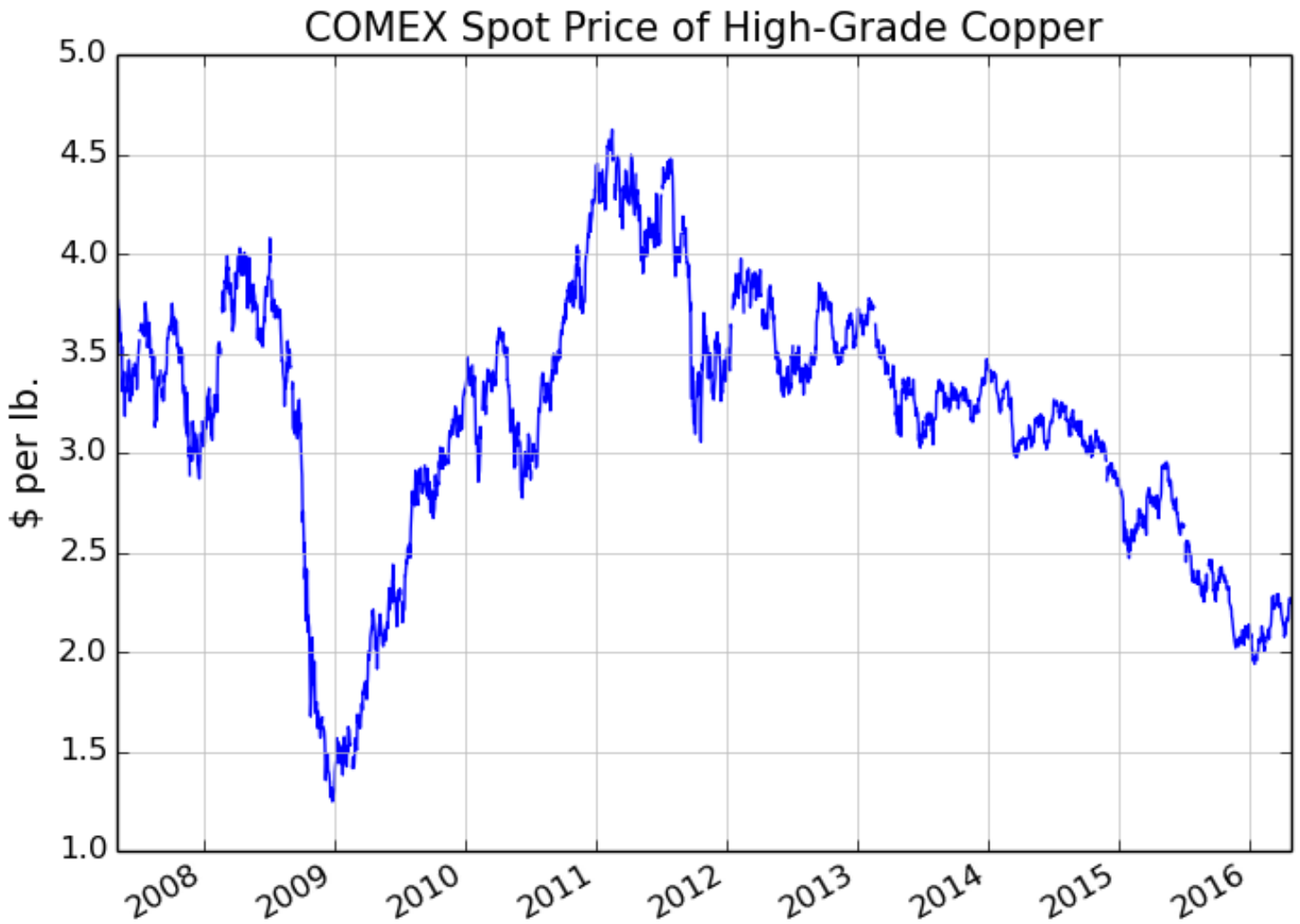
  Answer:

  In stock, the number of shares is the volume of a stock trade or investment. What is the equivalent unit in FX trading?

Introduction to Commodities

- These days, in practice, commodities are classified into vastly different sectors as follows:
  - Metals
    - Precious metals
    - Industrial metals
  - Energy
  - Agriculturals
    - Grains and Oilseeds
    - Softs
  - Livestock
  - Exotics
- Prices of commodities are *qualitatively* determined by the economics of supply and demand.
- From the financial trading and investment perspective, futures contracts are the dominant instruments for commodities.

Example of a Commodity (Data Source: WSJ)



Introduction to Fixed Income

- Fixed income or debt instruments are contracts between or among borrowers and lenders.
- Underpinning the fixed income instrument is CREDIT. It is the level of trust that the creditor or the lender has for the borrower.
- Without credit, it is virtually impossible to borrow money.
- Distinction of fixed income asset class is maturity or expiration.

Five-Year Coupon Bond Issued by U.S. Treasury

- New world order after World War II: The debt securities issued by the U.S. Treasury are considered to be virtually free of default risk.
- Treasury's coupon bond is a security that pays a coupon $(C/2)$ semi-annually with maturity T years with notional amount (principal) of A dollars.
- Present Value of Future Cash Flows:
  The present value (PV), i.e. the value today, is the sum of all the discounted future cash flows:

$$\begin{aligned}\text{PV} = &\frac{C/2}{(1+y/2)^1} + \frac{C/2}{(1+y/2)^2} + \cdots \\ &\cdots + \frac{C/2}{(1+y/2)^9} + \frac{C/2}{(1+y/2)^{10}} + \frac{A}{(1+y/2)^{10}}.\end{aligned}$$

Five-Year Coupon Bond Issued by U.S. Treasury (Cont'd)

- In this example, $T = 5$, and $y.$ is known as the yield to maturity.
- The yield to maturity is annualized. Since the U.S. Treasury pays coupon semi-annually, each half year's return is $y/2$.
- Like interest rate, coupon rate c is quoted on the annualized term. So $C = Ac$ is the dollar amount of coupon payment per year for A dollars invested.
- PV $\longleftrightarrow$ y

Pre-U Math Revision

You have learned geometric series in the Pre-U math. For $x \neq 1$,

$$\sum_{i=1}^{n} x^i = x\frac{1-x^n}{1-x}$$

Establish the validity (truth!) by a proof.

Treasury Coupon Bond in General

- For every dollar of face value and given the semi-annual payment convention, the present value p in percentages is

$$p = \frac{c}{2} \sum_{i=1}^{2n} \frac{1}{(1+y/2)^i} + \frac{1}{(1+y/2)^{2n}} \times 1. \tag{14}$$

This n-year par bond pays coupon at the fixed interest rate of c annually and is selling at $p = 1$.

- Being a geometric series, we rewrite the pricing formula (14) as

$$p = \frac{c}{y}\left(1 - \frac{1}{(1+y/2)^{2n}}\right) + \frac{1}{(1+y/2)^{2n}}.$$

Treasury Coupon Bond in General (Cont'd)

- Note that $p \approx c/y$ when n is large.
- After multiplying both sides by $(1+y/2)^{2n}$ and shifting the 1 obtained from the right side to the left side, the result is

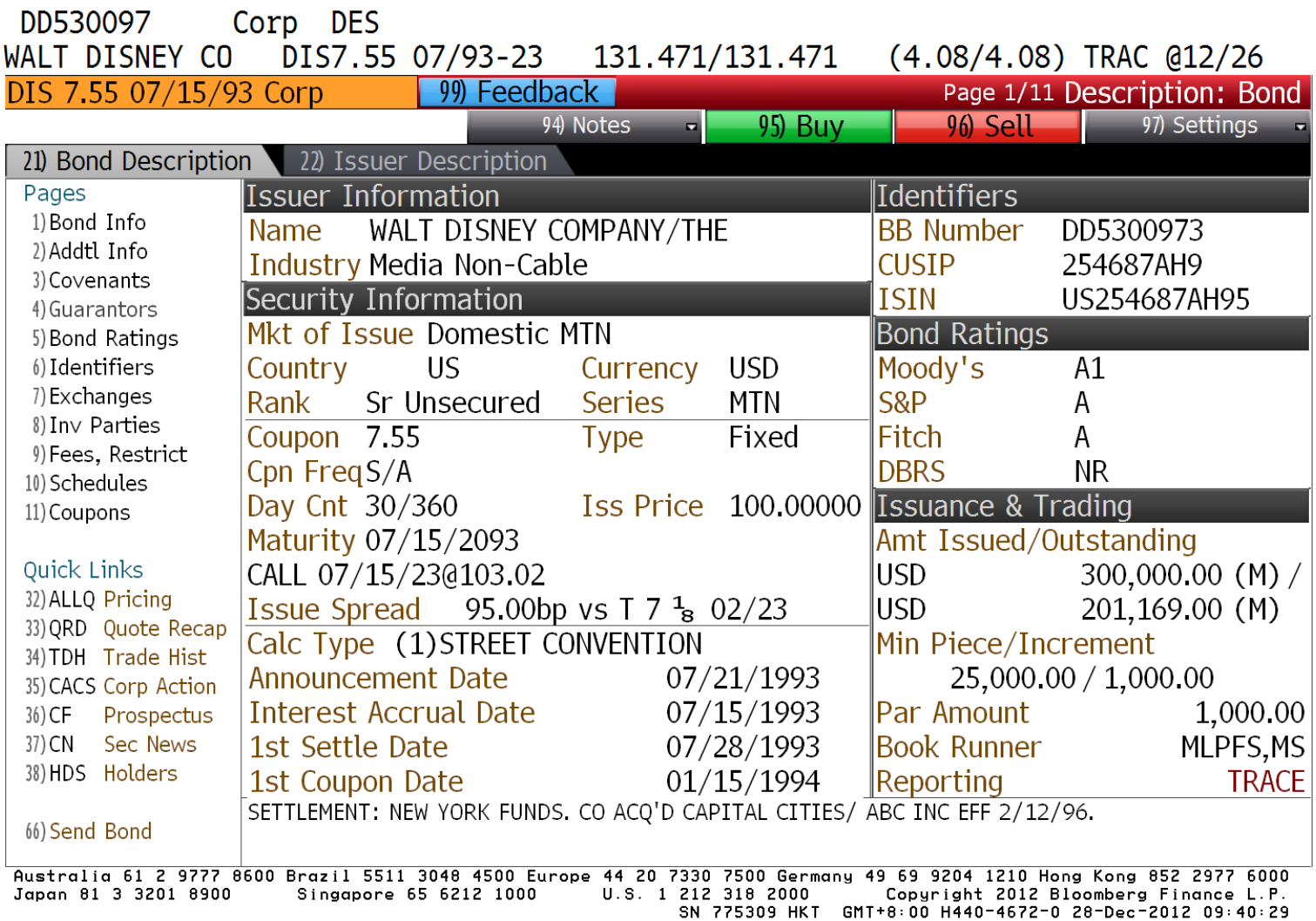
$$p(1+y/2)^{2n} - 1 = \frac{c}{y}\big((1+y/2)^{2n} - 1\big). \tag{15}$$

- When the bond is selling at par with $p = 1$, the coupon rate c and the yield to maturity y must be equal for this equation to hold.
- In other words, when a bond is selling at par, the coupon rate c *is* the yield to maturity y.

Capital Market Versus Money Market

- In the capital market, debt instruments of original maturities longer than 12 months are offered (e.g. 5-year bond)
- The money market is where short-term (up to 12 months) fixed income instruments are traded over the counter.
- The short-term (e.g. 3 months) U.S. Treasury bills are sold at public auctions every week, at a discount to their face value at maturity. Also known as the par amount, the face value is the amount paid by the U.S. Treasury at maturity in units of $1,000 to the investors.
- The bills are sold at a discount in the sense that the purchase price is less than the face value. The interest earned is simply the face value minus the purchase price.

Example of a Corporate Bond



Default

- All corporate bonds are defaultable.
- The biggest-ever U.S. bankruptcy is the collapse of Lehman Brothers in 2008, resulting in as much as $639 billion dollars of unpaid debts.
- According to the CRSP (Center for Research in Security Prices) database, 679 publicly listed companies domiciled in the U.S. went bust between 1931 to 2012.
- Over the recent 30 years from 1983 to 2012, the number of bankrupt companies is, on average, 19.17 per year.

Are U.S. Fixed Income Securities Really Risk-Free?

In Omaha, the U.S. is still triple A. In fact, if there were a quadruple-A rating, I'd give the U.S. that.

Picture source: USA Today

Other Investments

- Real Estate
- Collectables
  - Paintings
  - Sculptures
  - Jewellery
  - Antiques
- Venture Capital

Picture source: Wikipedia

Takeaways

- Long versus Short
- Absolute versus Relative

$$\text{P\&L} = \text{Units} \times (\text{selling price} - \text{buying price})$$

$$\text{Return} = \frac{\text{selling price} - \text{buying price}}{\text{buying price}}$$

- Arithmetic versus Geometric
- Capital Market versus Money Market
- Yield to Maturity versus Bond Price
- Defaultable versus Risk-Free

Week 2 Assignment from Chapter 2

Exercises

Questions 1 to 4, Question 6, and Question 9

Additional Exercises

Question 5, Question 10